QCD Phase Transition and Hadron Bubble Formation in the Dual Ginzburg-Landau Theory

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Abstract

We study the QCD phase transition at finite temperature and discuss the hadron bubble formation in the dual Ginzburg-Landau theory, which is an effective theory of QCD and describes the color confinement via QCD-monopole condensation. We formulate the effective potential at various temperatures and find that thermal effects reduce the QCD-monopole condensate and bring a first-order deconfinement phase transition. Based on this effective potential at finite temperature, we investigate properties of hadron bubbles created in the early Universe and discuss the hadron bubble formation process.

I. INTRODUCTION

At low temperature and low density, colored particles, quarks and gluons are confined in hadrons due to the nonperturbative nature of the QCD vacuum. However, as the temperature increases above a certain critical temperature, these colored particles would be liberated like free particles and the vacuum becomes quark gluon plasma (QGP) phase [1]. This phenomenon is called QCD phase transition and many physists try to create QGP in high-energy heavy-ion collisions (e.g. RHIC at Brookhaven). After two heavy ions collide, and pass through each other, the huge energy deposition at central region leads to QGP [1]. The QCD phase transition also happened as a real event in the early Universe, which strongly influenced the afterward nucleosynthesis. Thus, the QCD phase transition is interesting in the various fields.

The QCD phase transition is caused by the change of QCD vacuum. In low energy region, the QCD vacuum is regarded as the dual version of superconductor [2]. Condensation of the magnetic charge makes the color-electric field excluded from the QCD vacuum, which leads to the color confinement. This picture is constructed after abelian gauge fixing as 't Hooft proposed [3]. The most relevant gauge for the discussion of confinement is the abelian gauge, where QCD-monopoles appear as topological objects having the magnetic charges. This is modeled in the dual Ginzburg Landau (DGL) theory [4,5], which has a strong connection with QCD and are supported from the recent results of lattice QCD [6].

II. DUAL GINZBURG-LANDAU THEORY

The DGL lagrangian [5,7] in the pure gauge system is written by using the dual gauge field, $B_\mu = \vec{B}_\mu \cdot \vec{H} = B_\mu^3 T^3 + B_\mu^8 T^8$, and the QCD-monopole field, $\chi = \sum_a \sqrt{2}\chi_a E_a$ ($E_1 = \frac{1}{\sqrt{2}}(T_6 + iT_7), E_2 = \frac{1}{\sqrt{2}}(T_4 - iT_5), E_3 = \frac{1}{\sqrt{2}}(T_1 + iT_2)$);

$$\mathcal{L}_{DGL} = \text{tr}\hat{\mathcal{L}}$$

$$\hat{\mathcal{L}} = -\frac{1}{2}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 + [\hat{D}_\mu, \chi]^\dagger [\hat{D}^\mu, \chi] - \lambda(\chi^\dagger \chi - v^2)^2, \tag{1}$$

where $\hat{D}_\mu = \hat{\partial}_\mu + igB_\mu$ is the dual covariant derivative. The dual gauge field B_μ is defined on the dual gauge manifold $U(1)_3^m \times U(1)_8^m$, which is the dual space of the maximal torus subgroup $U(1)_3^e \times U(1)_8^e$ embedded in the original gauge group SU(3) [5]. The abelian field strength tensor is written as $F_{\mu\nu} = {}^*(\partial \wedge B)_{\mu\nu}$ so that the role of the electric and magnetic field are interchanged in comparison with the ordinary A_μ description. The QCD-monopole has magnetic charge $g\vec{\alpha}$, where $\vec{\alpha}$ is the root vector, and g is the dual gauge coupling constant. If the QCD-monopole condenses, $|\chi_\alpha| = v$, the dual gauge field acquires mass $m_B = \sqrt{3}gv$ by the dual Higgs mechanism and the color-electric field is excluded from the QCD vacuum. In the DGL theory, QCD-monopole condensate is the order parameter of the deconfinement phase transition [7].

III. QCD PHASE TRANSITION

To investigate the QCD phase transition at finite temperature, we formulate the effective potential [7](thermodynamical potential) as the function of QCD-monopole condensate $|\chi_\alpha| = \bar{\chi}$,

$$V_{\text{eff}}(\bar{\chi}; T) = 3\lambda(\bar{\chi}^2 - v^2)^2 + 3\frac{T}{\pi^2}\int_0^\infty dk k^2 \ln\left(1 - e^{-\sqrt{k^2+m_B^2}/T}\right)$$

$$+ \frac{3}{2}\frac{T}{\pi^2}\int_0^\infty dk k^2 \ln\left(1 - e^{-\sqrt{k^2+m_\chi^2}/T}\right), \tag{2}$$

where $m_B = \sqrt{3}g\bar{\chi}$ and $m_\chi = 2\sqrt{\lambda}\bar{\chi}$. The effective potential is plotted in Fig.1 and the behavior of the QCD-monopole condensate at various temperatures is shown in Fig.2. As the temperature increases, the QCD-monopole condensate in the physical vacuum, $\bar{\chi}_{\text{phys}}(T)$, decreases and the broken dual gauge symmetry tends to be restored. A first order phase transition is found at the critical temperature, $T_c \simeq 0.49$ GeV. This phase transition is regarded as the deconfinement phase transition, because there is no confining force among colored particles in the QCD vacuum with $\bar{\chi}_{\text{phys}}(T) = 0$. The critical temperature $T_c = 0.49$ GeV, however, seems much larger than the recent lattice QCD prediction [8,9]. We introduce therefore the temperature dependence on the parameter λ as,

$$\lambda(T) = \lambda\left(\frac{T_c - aT}{T_c}\right), \tag{3}$$

according to the asymptotic freedom property of QCD. The behavior of the QCD-monopole condensate is shown in Fig.3 for the case where the parameter a are chosed so as to provide the critical temperature at T_c=0.2GeV. We find a weak first order phase transition.

Using this QCD-monopole condensate at various temperatures, we next consider the string tension of hadron (Fig.4) and the glueball mass (Fig.5). As the temperature increases, the string tension becomes smaller and drops rapidly near the critical temperature, which agrees with the lattice simulation data [10]. This means that the hadron size at finite temperature becomes larger as the temperature increases. Here, we find also the large reduction of the glueball mass [7] near the critical temperature, where the glueball excites violently as its mass becomes small and it leads to the QCD phase transition.

IV. HADRON BUBBLE FORMATION IN EARLY UNIVERSE

Finally, we consider the application of the DGL theory to big bang. As Witten proposed [11], if the QCD phase transition is of first order, the hadron and the QGP phase should coexist in early Universe. Such a mixed phase may cause the inhomogeneity of the Universe in the baryon number distribution. This inhomoginity affects the primordial nucleo-synthesis [12].

As a result of the 1st order phase transition, hadron bubbles appear in the QGP phase near the critical temperature. We now consider how hadron bubbles are formed in the DGL theory. In the supercooling system, the free energy of the hadron bubble with radius R profile $\bar{\chi}(r; R)$ is written using the effective potential at finite temperature,

$$E[\bar{\chi}(r; R)] = 4\pi \int_0^\infty dr r^2 \{3(\frac{d\bar{\chi}(r; R)}{dr})^2 + V_{\text{eff}}(\bar{\chi}; T)\}. \tag{4}$$

We use the sine-Gordon kink ansatz for the profile of the QCD-monopole condensate,

$$\bar{\chi}(r; R) = \bar{\chi}_H \tan^{-1} e^{(R-r)/\delta} / \tan^{-1} e^{R/\delta}, \tag{5}$$

where the thickness of the surface δ is determined by the free energy minimum conditions. The result is shown in Fig.6. The QCD-monopole condensate $\bar{\chi}(r; R)$ is connected smoothly between inside and outside the bubble. The energy density of the hadron bubble is shown in Fig.7. It is negative inside and positive near the boundary surface. The total energy is roughly estimated as the sum of the surface term (corresponding to the positive region near the surface) and the volume term (corresponding to the negative region inside the bubble).

The energy of the hadron bubble with radius R is shown in Fig.8. The bubble whose radius is smaller than critical radius R_c collapses. Only larger bubbles ($R > R_c$) are found to grow up from the energetical argument. However, the creation of large bubbles is suppressed because of formation probability. In the bubble formation process, there exists a large barrier height h of the effective potential and therefore the creation of large bubbles needs the large energy fluctuation above the barrier height. Such a process is suppressed because of the thermal dynamical factor (proportional to bubble formation rate), $\exp(-\frac{4}{3}\pi R_c^3 h/T)$. Thus, the only small bubbles are created practically, although its radius should be larger than R_c energitically. The temperature dependence of the critical radius and the bubble formation rate is shown in Fig.9 and Fig.10 respectively. In the temperature region of the supercooling

state, i.e, $T_{\text{low}} < T < T_{\text{c}}$, the hadron bubbles are created. As the temperature decreases, the size of hadron bubble becomes smaller, but the bubble formation rate becomes larger.

From these results, we can imagine how the QCD phase transition happens in the big bang scenario [13]. At the first stage slightly below T_c, only large bubbles are created but its rate is quite small. As temperature is lowered, smaller bubbles are created with much formation rate. During this process, the created hadron bubbles expand with radiating shock wave which reheats QGP phase [13]. Near T_{low} many small bubbles are violently created. Finally QGP phase is isolated like the bubble [13]. Such an evolution of the hadron bubble can be obtained from the numerical simulation using the DGL theory.

We study the QCD phase transition at finite temperature in the DGL theory. Thermal effect reduces the QCD-monopole condensate, and the QCD vacuum is changed into the QGP phase. According to the reduction of the QCD-monopole condensate at high temperature, the string tension becomes smaller, the hadron size becomes larger and glueball mass becomes smaller. We apply the DGL theory to the hadron bubble formation in early Universe. Using the effective potential, we estimate the size of hadron bubble at various temperatures.

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Figure Caption

Fig.1 The effective potential at various temperatures as a function of the QCD-monopole condensate. The minima of $V_{\text{eff}}(\bar{\chi}; T)$ are plotted by $\times$.

Fig.2 The QCD-monopole condensate $\bar{\chi}_{\text{phys}}(T)$ at the minimum of the effective potential as a function of temperature T.

Fig.3 The QCD-monopole condensate $\bar{\chi}_{\text{phys}}(T)$ at the minimum of the effective potential as a function of the temperature in the case of variable $\lambda(T)$, which reproduces T_c=0.2GeV.

Fig.4 The string tensions $k(T)$ for a constant λ and a variable $\lambda(T)$ as functions of the temperature T. The lattice QCD results [10] in the pure gauge are shown by black dots near and below the critical temperature.

Fig.5 The masses m_B and m_χ of the dual gauge field B_μ and the QCD-monopole field χ as the function of temperature.

Fig.6 The profile of the QCD-monopole condensate in the hadron bubble. Outside of bubble, QCD-monopole does not condense (QGP phase), while inside the bubble, QCD-monopole condenses (hadron phase).

Fig.7 The energy density of the hadron bubble. It is negative inside and positive near the boundary surface.

Fig.8 The total energy of the bubble is plotted as a function of the hadron bubble radius$R$. The total energy is roughly estimated as the sum of the volume term and the surface term.

Fig.9 The critical radius R_c, corresponding to maximum of the energy in Fig.8, is plotted as a function of temperature.

Fig.10 The dominant factor of the bubble formation rate $P(R_c; T) \equiv exp(-4\pi R_c^3 h(T)/3T)$ is plotted as a function of temperature. As the temperature decreases, the radius of the created hadron bubble becomes smaller, but the bubble formation rate larger.

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